Total Marks - 84
Attempt Questions 1 – 7
All questions are equal value.
Answer each question in a SEPARATE writing booklet.

Question 1 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the exact value of $\int_0^{\frac{\pi}{6}} \sec^2 2x\,dx$. **2**

(b) Solve the inequation $\frac{x}{x-3} \geq 2$, $x \neq 3$. **3**

(c) Find the acute angle between the lines $x - y + 3 = 0$ and $2x + y + 1 = 0$ **2**
Give your answer correct to the nearest minute.

(d) Find the co-ordinates of P, the point which divides the interval $\mathrm{A}(-5,-3)$ and $\mathrm{B}(4,-6)$ externally in the ratio $2:3$. **2**

(e) When $P(x) = x^3 + 3x^2 - mx + n$ is divided by $(x+2)$ the remainder is 9 and when $P(x)$ is divided by $(x-3)$ the remainder is 49. **3**
Find the values of m and n.

Question 2 (12 Marks) Use a SEPARATE writing booklet.

(a) Sketch the graph of $y = \cos^{-1}\left(\frac{x}{2}\right)$ **2**
Your graph must indicate the domain and range.

(b) Find $\frac{d}{dx}\left(x sin^{-1} 5x\right)$ **2**

(c) (i) Write $2\cos\theta + \sin\theta$ in the form $A\cos(\theta - \alpha)$ where A>0 and $0 \leq \alpha \leq \frac{\pi}{2}$. **2**

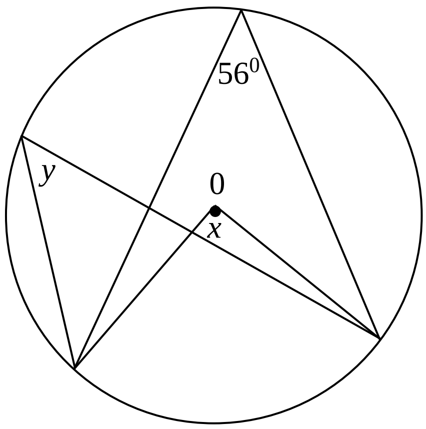
(ii) Hence, or otherwise solve the equation $2\cos\theta + \sin\theta = \sqrt{5}$ for $0 \leq \theta \leq 2\pi$. **2**

Give your answer correct to 2 decimal places.

Question 2 Continued. **Marks**

(d) Find the general solution for $\cos\theta = \frac{\sqrt{3}}{2}$. **2**

(e) O is the centre of the circle. Find the value of the pronumerals, *x* and *y*, giving reasons. **2**



Question 3 (12 Marks) Use a SEPARATE writing booklet.

(a) Use the Binomial Theorem to find the term independent of x in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{18}$. **3**

(b) Given $f(x) = \cos x - \ln x$

(i) Show that a root of $f(x) = 0$ lies between 0.5 and1.5. **1**

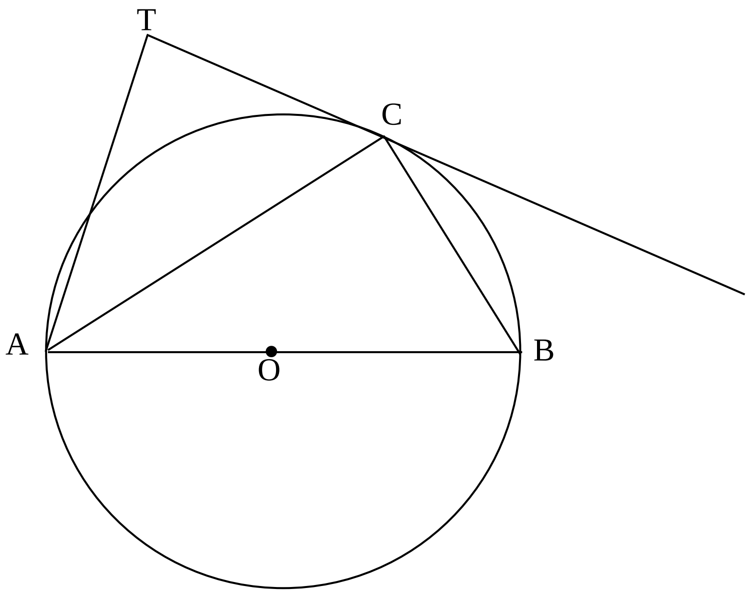
(ii) Use Newton's Method once to find an approximation to this root of the equation, correct to 2 decimal places, starting with $x = 1$. **2**

(c) Evaluate $\int_0^{0.4} \frac{3dx}{4+25x^2}$ **3**

Questions continues on Page 3.

Question 3 Continued. **Marks**

(d) AOB is the diameter of a circle, centre O. C is the point of contact of the tangent, TC such that AC bisects $\angle$TAB. Prove that AT is perpendicular to TC. **3**



Question 4 (12 Marks) Use a SEPARATE writing booklet.

(a) Show by Mathematical Induction that $9^n - 7^n$ is divisible by 2, for $n \geq 1$. **4**

(b) How many different arrangements of the letters of the word NEWINGTON are possible? **2**

(c) (i) A committee of eight people is to be formed from a group of twenty people. In how many different ways can the committee be formed? **1**

(ii) This group consists of twelve men and eight women, how many ways can committee of four men and four women be formed if Kate and Pete must be included? **2**

(iii) In how many ways can these 8 committee members sit around a circular table with no conditions restricting where anyone sits? **1**

(iv) If the men and women alternate where they sit, how many arrangements are possible? **2**

Question 5 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) P $\left(2ap, ap^2\right)$ and Q $\left(2aq, aq^2\right)$ are two points on the parabola $x^2 = 4ay$.

The tangents at P and Q meet at T which is always on the parabola $x^2 = -4ay$.

(i) Given the equation of the tangent at P is $y - px + ap^2 = 0$. **2**

Show that T is the point $\left(a\left(p+q\right), apq\right)$.

(ii) Show that $p^2 + q^2 = -6pq$. **1**

(iii) Find M, the midpoint of PQ. **1**

(iv) Find the equation of the locus of M. **3**

(b) A casserole is cooling in a room that has a constant temperature of C. At a given time, t minutes the temperature decreases according to $\frac{dT}{dt} = -k(T - 22)$ where k is a positive constant.

(i) Show that $T = 22 + Ae^{-kt}$ is a solution to the equation. **1**

(ii) Given that the initial temperature of the casserole is $80^0 C$ and it cools to $60^0 C$ after 10 minutes. Find A and k. **2**

(iii) How long will it take for the casserole to cool to $30^0 C$? **2**

Question 6 (12 Marks) Use a SEPARATE writing booklet.

(a) Ben fires an arrow horizontally with a speed of 60 ms^{-1} from the top of a 20 m high cliff. Use $g = 10m/s^2$.

(i) Show that the position of the arrow at time, t seconds is given by $x = 60t$ and $y = 20 - 5t^2$. **2**

(ii) Find the time taken for the arrow to hit the ground. **1**

(iii) Find the distance the arrow is from the base of the cliff when it hits the ground. **1**

(iv) Find the acute angle to the horizontal at which the arrow hits the ground. **2**

(b) A particle's motion is defined by the equation, $v^2 = 12 + 4x - x^2$ where x is its displacement from the origin in metres and v is its velocity in ms^{-1}. Initially, it is 6 metres to the right of the origin.

(i) Show that the particle is moving in Simple Harmonic Motion. **1**

(ii) Find the centre, the period and the amplitude of the motion. **3**

(iii) The displacement of the particle at any time is given by the equation $x = a\sin(nt + \theta) + b$. **2**

Find the values of, θ and b given $0 \le \theta \le 2\pi$.

Question 7 (12 Marks) Use a SEPARATE writing booklet.

(a) Using the substitution $u = 2x + 1$ find $\int_0^1 \frac{4x}{2x+1} dx$. **4**

(b) For $(a+b)^n$ the general term is $T_{k+1} = {}^nC_k a^{n-k} b^k$.

For the following expansion $(3+11x)^{19}$:

(i) Show that $\frac{T_{k+1}}{T_k} = \frac{11x(20-k)}{3k}$. **2**

(ii) Hence find the greatest coefficient of $(3+11x)^{19}$. **3**

(c) (i) Use the binomial theorem to obtain an expansion for $(1+x)^{2n} + (1-x)^{2n}$ where n is a positive integer. **1**

(ii) Hence evaluate $1 + {}^{20}C_2 + {}^{20}C_4 + \dots\dots\dots + {}^{20}C_{20}$. **2**

END OF PAPER

(1) a) $\int_0^{\pi/6} \sec^2 2x\,dx = \left[\frac{\tan 2x}{2}\right]_0^{\pi/6} = \frac{\tan\frac{\pi}{3} - \tan 0}{2} = \underline{\frac{\sqrt{3}}{2}}$ (1) Correct Integral (1) Correct Answer

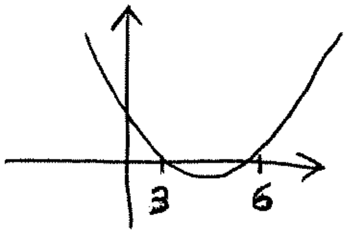
b) $\frac{x}{x-3} \geq 2 \qquad \times (x-3)^2 \quad (x-3)x \geq 2(x-3)^2$

$x^2 - 3x \geq 2x^2 - 12x + 18$ (1)

$0 \geq x^2 - 9x + 18$

$0 \geq (x-6)(x-3)$ (1)

$3 < x \leq 6$ (1) Answer $x \neq 3$.



c) $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad x - y + 3 = 0 \quad y = x + 3 \quad m_1 = 1$

$2x + y + 1 = 0 \quad y = -2x - 1 \quad m_2 = -2$ (1)

$\tan\alpha = \left|\frac{1+2}{1-2}\right| = 3$ (1) $\qquad \alpha = 71^\circ 33' 54\ldots = \underline{71^\circ 34'}$ (nearest minute)

d) $A(-5, -3) \quad B(4, -6)$ External Division $2 : -3$

$x = \frac{mx_2 + nx_1}{m+n} = \frac{8+15}{-1} = -23$ (1) $\quad y = \frac{my_2 + ny_1}{m+n} = \frac{(2 \times -6) + (-3 \times -3)}{2-3} = \frac{-3}{-1} = 3$

P has co-ordinates $\underline{(-23, 3)}$ (1)

e) $P(x) = x^3 + 3x^2 - mx + n$.

$P(-2) = 9 \qquad -8 + 12 + 2m + n = 9 \qquad 2m + n = 5$ (A) (1)

$P(3) = 49 \qquad 27 + 27 - 3m + n = 49 \qquad -3m + n = -5$ (B) (1)

(A) − (B) $5m = 10 \quad \therefore \underline{m = 2} \quad \therefore \underline{n = 1.}$ (1)

MARKERS COMMENTS

a) Generally well done. Some gave coefficient of 2 not ½

b) Most students knew to multiply by square of denominator. Mistakes came collecting terms, factorising & interpreting region. Students were not penalised for failure to recognise $x \neq 3$.

c) Many recalled the formula incorrectly

d) Students were poor at applying standard formula. Most common error was $mx_1 + nx_2$ in numerator.

e) Students recognising factor theorem generally completed this question well.

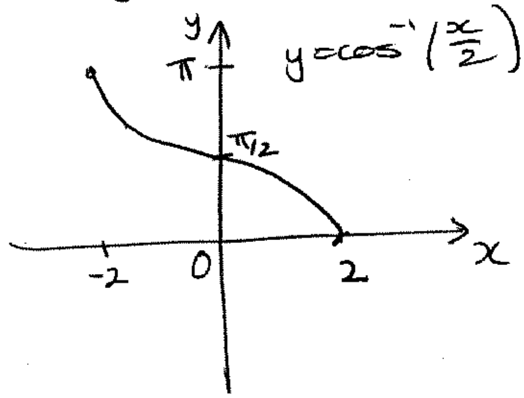
(a) Sketch $y = \cos^{-1}\left(\frac{x}{2}\right)$

1 mark correct curve
1 mark correct domain and range.

Domain $-1 \leq \frac{x}{2} \leq 1$

$-2 \leq x \leq 2$

Range $0 \leq y \leq \pi$



(b) $\frac{d}{dx}\left(x \sin^{-1} 5x\right)$

$= \sin^{-1} 5x + x \times \frac{5}{\sqrt{1-25x^2}}$

1 mark correct differentia[tion] of $\sin^{-1} 5x$

$= \sin^{-1} 5x + \frac{5x}{\sqrt{1-25x^2}}$

1 mark correct answer from their working

(c) $2\cos\theta + \sin\theta = A\cos(\theta - \alpha)$ $A > 0$

$0 \leq \alpha \leq \frac{\pi}{2}$.

$A = \sqrt{2^2 + 1^2}$

$A = \sqrt{5}$

1 mark $A = \sqrt{5}$

OR

$\therefore 2\cos\theta + \sin\theta = \sqrt{5}\cos(\theta - \alpha)$

and $\tan\alpha = \frac{1}{2}$

$\alpha = 0.463647609^c$

$\alpha = 0.46^c$ (2dp)

1 mark $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

or $\alpha = 0.463647\ldots$

$\alpha = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$

$\alpha = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\therefore 2\cos\theta + \sin\theta = \sqrt{5}\cos(\theta - 0.46^c)$

1 mark correct solution.

OR accept $= \sqrt{5}\cos(\theta - \alpha)$ $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

degrees not accepted.

(c)(ii) $2\cos\theta + \sin\theta = \sqrt{5}$

$\therefore \sqrt{5}\cos(\theta - 0.46^c) = \sqrt{5}$

$\therefore \cos(\theta - 0.46^c) = 1$ 1 mark correct equati

$\therefore \theta - 0.46^c = 0, 2\pi, 4\pi, \ldots$

But $0 \le \theta \le 2\pi$

$\theta = 0.46^c$ (2dp) 1 mark correct θ.

(d) $\cos\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \cos\frac{\pi}{6}$ 1 mark for $\frac{\pi}{6}$

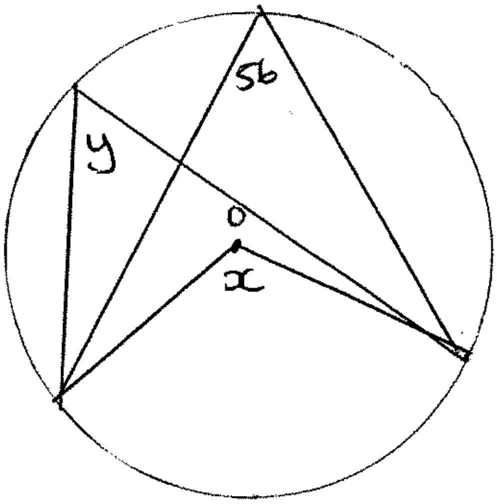
general solution

$\theta = 2n\pi \pm \alpha$

$\theta = 2n\pi \pm \frac{\pi}{6}$ 1 mark correct answer only.

OR $\theta = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}, \ldots$

(e)



$x = 2 \times 56°$ (angle at the centre and the circumference)

$x = 112°$

$y = 56°$ (angles on the same arc)

(angles on the same chord)

(angles in the same segment)

1 mark each correct value with correct reason.

Solutions Ext. 1 Trial HSC 2006

Question 3

(a) Independent term $\Rightarrow x^0$

$$(x^2 - \tfrac{1}{2}x^{-1})^{18} \Rightarrow {}^{18}C_r \, (x^2)^{18-r} \, (-\tfrac{1}{2})^r (x^{-r})$$

$$\therefore \quad = {}^{18}C_r \, x^{36-2r} \, . \, x^{-r} \, (-\tfrac{1}{2})^r$$

$$= {}^{18}C_r \, (-\tfrac{1}{2})^r \, x^{36-3r}$$

$\therefore \; 36 - 3r = 0$

$\therefore \; r = 12$

$\therefore$ Independent term $= {}^{18}C_{12} \, (\tfrac{1}{2})^{12}$

$= \dfrac{4641}{1024}$

3 mks correct ans.
2 mks correct method for finding r
1 mk correct expansion

(b) (i) For $f(x) = \cos x - \ln x$ [1 mk]

$f(x)$ is cts for $0.5 \le x \le 1.5$

$f(0.5) > 0$ ie. $f(0.5) \simeq 1.57$

$f(1.5) < 0$ ie $f(1.5) \simeq -0.33$

$\therefore$ root lies in the interval $0.5 \le x \le 1.5$

(ii) By Newton's Method:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \quad ; \text{ if } x_0 = 1$$

then $x_1 = 1 - \dfrac{f(1)}{f'(1)}$.

$f'(x) = -\sin x - \dfrac{1}{x}$ $\quad \therefore \; f'(1) = -\sin(1) - 1$.

$f(x) = \cos x - \ln x$ $\quad \therefore \; f(1) = \cos 1$

$\therefore \; x_1 = 1 + \dfrac{\cos 1}{\sin(1)+1}$.

$\therefore \; x_1 \doteqdot 1.29$ (2 dp).

1 mk correct use of formula
2 mks correct ans

(c) $\displaystyle\int_0^{0.4} \frac{3\,dx}{4+25x^2} = 3\int_0^{0.4} \frac{1}{4+25x^2}\,dx$

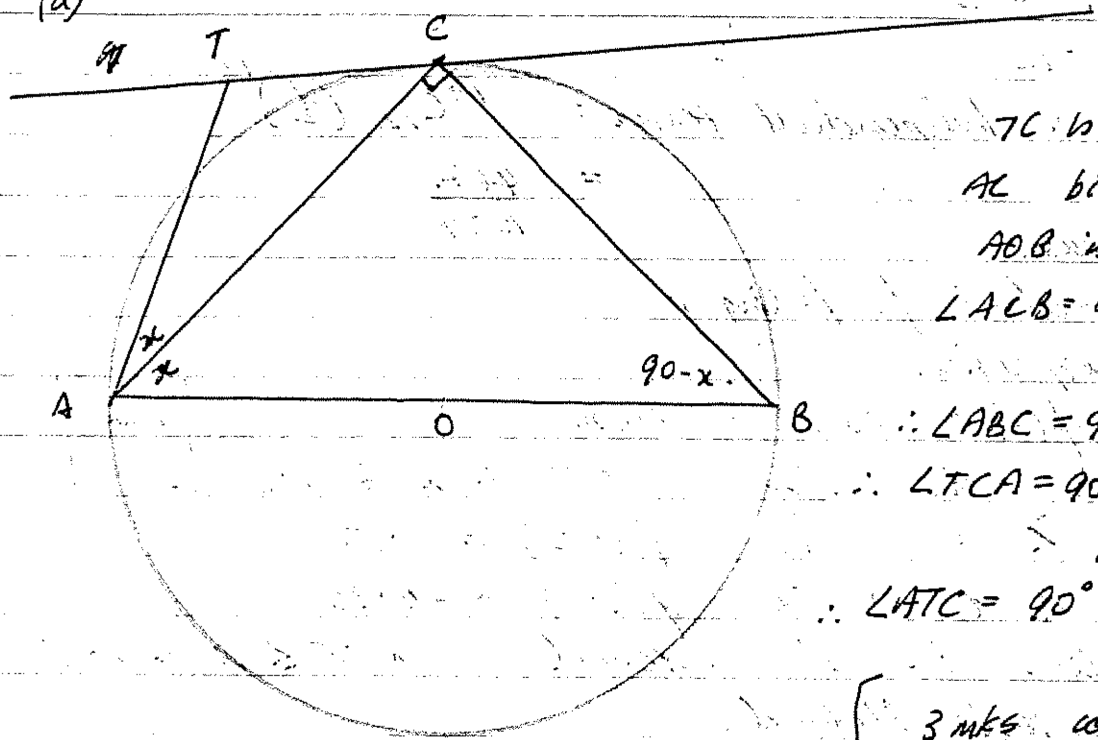
$= \frac{3}{2\times 5}\left[\ \cdot\ \tan^{-1} \frac{5x}{2}\right]_0^{0.4}$

$= 0.3\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$= \frac{3\pi}{40}$.

[3 mk correct ans
2 mk correct integral
1 mk correct integration method]

(d)



TC is a tangent
AC bisects $\angle TAB$.
AOB is a diameter.
$\angle ACB = 90°$ (angle in a semi-circle).
$\therefore \angle ABC = 90 - x$.
$\therefore \angle TCA = 90 - x$ (angle in alternate segment)
$\therefore \angle ATC = 90°$ (angle sum of ΔATC)

[3 mks correct method
~~2 mks~~ 1 mk (angle in semi)
1 mk (angle in alt seg).
1 mk (angles in Δ).]

Question 4

a) Step 1 ~~Test~~ Prove true for $n = 1$

$9^1 - 7^1 = 2$ $\therefore$ divisible by 2 ✓

Step 2 Assume true for $n = k$

$9^k - 7^k = 2M$ where M is a positive integer

Step 3 Prove true for $n = k+1$

$$9^{k+1} - 7^{k+1} = 9.9^k - 7.7^k$$ ✓

$$= 7(9^k - 7^k) + 2.9^k$$ ✓

$$= 7(2m) + 2.9^k \text{ (From Step 2)}$$

$$= 2(7m + 9^k)$$ ✓

$\therefore 9^{k+1} - 7^{k+1}$ is divisible by 2

Step 4

b) $\frac{9!}{3!} = 60480$ ✓ ✓

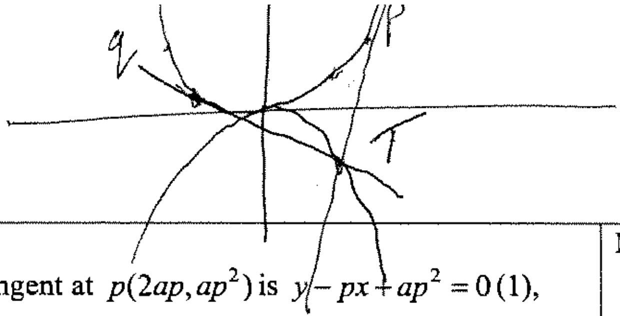
c) i) ${}^{20}C_8 = 125970$ ✓

ii) ${}^{11}C_3 \times {}^{7}C_3 = 5775$ ✓ ✓

iii) $7!$ ✓ $= 5040$

iv)

$4! \times 3! = 144$ ✓ ✓



5a)		Marking Criteria
	(i) As the tangent at $p(2ap, ap^2)$ is $y - px + ap^2 = 0\,(1)$, therefore the tangent at Q is $y - qx + aq^2 = 0\,(2)$. Solving (1) and (2) simultaneously: (2) − (1) $(p-q)x - a(p^2-q^2) = 0$	1 mark process
	$\therefore\; x = \dfrac{a(p^2-q^2)}{(p-q)} = a(p+q)$ $\therefore\; y = p \times a(p+q) - ap^2 = apq$ $\therefore\; T\,(a(p+q), apq)$	1 mark for correct working
	(ii) As T lies on $x^2 = -4ay$ $\therefore (a(p+q))^2 = -4a \times apq$ $\therefore p^2 + 2pq + q^2 = -4pq$ $\therefore p^2 + q^2 = -6pq$	1 mark for correctly using given information
	(iii) Mid-point of PQ: $M\left(\dfrac{2ap+2aq}{2}, \dfrac{ap^2+aq^2}{2}\right) = M\left(a(p+q), \dfrac{a}{2}(p^2+q^2)\right)$	1 mark for correct working
	(iv) Let $x = a(p+q),\quad y = \dfrac{a}{2}(p^2+q^2)$ $\therefore\; y = \dfrac{a}{2}\left((p+q)^2 - 2pq\right)$ $\therefore\; y = \dfrac{a}{2}\left((\dfrac{x}{a})^2 - 2pq\right)$ $\therefore\; y = \dfrac{x^2}{2a} - apq$	1 mark for starting with the correct parameters!
	$\therefore\; y = \dfrac{x^2}{2a} + \dfrac{a(p^2+q^2)}{6}$ $\therefore\; y = \dfrac{x^2}{2a} + \dfrac{a(p^2+q^2)}{2} \times \dfrac{1}{3}$ $\therefore\; y - \dfrac{y}{3} = \dfrac{x^2}{2a}$	1 mark for correct process
	$\therefore\; x^2 = \dfrac{4a}{3}y$ Locus of M is a parabola.	1 mark for correct answer

<table>
<tr><td>5b)</td><td>

Given $\frac{dT}{dt} = -k(T-22)$

(i)

$$T = 22 + Ae^{-kt}$$
$$\frac{dT}{dt} = -kAe^{-kt}$$
$$\frac{dT}{dt} = -k(T-22) \quad as \quad Ae^{-kt} = T-22$$

(ii) Now,
$when \quad t=0, \quad T=80°$
$\therefore 80 = 22 + A\times 1$
$\therefore A = 58$
And
$when \quad t=10, \quad T=60°$
$\therefore 60 = 22 + 58\times e^{-k\times 10}$
$\therefore e^{-10k} = \frac{19}{29}$
$\therefore k = -\frac{1}{10}\ln\left(\frac{19}{29}\right)$

(iii)
As
$T = 22 + 58e^{\frac{t}{10}\left(\ln\frac{19}{29}\right)}$
$when \quad T = 30$
$\frac{4}{29} = e^{\frac{t}{10}\left(\ln\frac{19}{29}\right)}$
$\therefore \frac{t}{10} = \frac{\ln\left(\frac{4}{29}\right)}{\ln\frac{19}{29}}$
$\therefore t \approx 46.84 \text{ min}$ (46 min 51 sec)

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Marking Criteria

1 mark for correct working

1 mark for correct value of A

1 mark for correct value of k

1 mark for correct process

1 mark for correct answer

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Q6

(i) **Horizontally**

$\ddot{x} = 0$

$\therefore \dot{x} = t + c$

when $t=0$, $\dot{x} = 60$

$\therefore \dot{x} = 60$

$\therefore x = 60t + c$

when $x=0$, $t=0$

$\therefore \boxed{x = 60t}$

Vertically

$\ddot{y} = -g$
$= 10$

$\dot{y} = -10t + c$

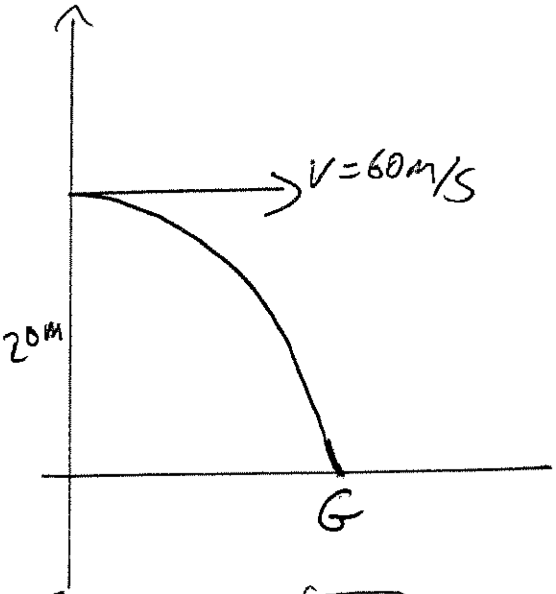
when $t=0$, $\dot{y}=0$

$\therefore \dot{y} = -10t$

$\therefore y = \frac{-10t^2}{2} + c$

when $t=0$ $y=20$

$\therefore \boxed{y = 20 - 5t^2}$



2

(ii) Arrow hits the ground when $y=0$

$\therefore 20 - 5t^2 = 0$

$(2-t)(2+t) = 0$

$\therefore$ after 2 seconds $[t \geq 0]$

1

(iii) when $t = 2$

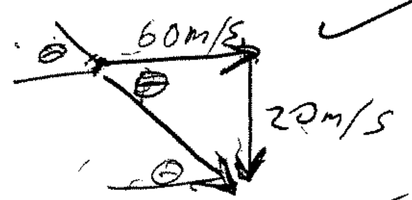
$x = 60 \times 2$

$= 120$ m

1

(iv) at G $\dot{x} = 60$ m/s

$\dot{y} = -20$ m/s



$\tan\theta = \frac{20}{60}$

$\theta = \tan^{-1}\left(\frac{1}{3}\right)$

accept $\theta = \underline{18^\circ 26'}$

2

(b) $v^2 = 12 + 4x - x^2$

(i) $\frac{1}{2}v^2 = 6 + 2x - \frac{1}{2}x^2$ 1

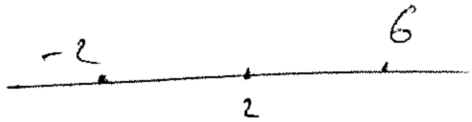
$\frac{d[\frac{1}{2}v^2]}{dx} = 2 - x$ $\therefore$ SHM as $\ddot{x}$ is directly proportional to its displacement.

$= -1^2(x-2)$

$n = 1$

(ii) $v^2 = (6-x)(2+x)$

$v = \pm\sqrt{(6-x)(2+x)}$



Centre of Motion: $x = 2$

Period $= \frac{2\pi}{1} = 2\pi$ sec 3

Amplitude $= 4$ m

(iii) $x = a\sin(nt + \theta) + b$

$x = 6, \ a = 4 \ n = 1, \ t = 0$

$6 = 4\sin\theta + b$

now $\dot{x} = an\cos(nt + \theta)$

$\dot{x} = 0$ when $t = 0$

$\therefore \cos\theta = 0$ 2

$\theta = \frac{\pi}{2}$ #

$\therefore 6 = 4\sin\frac{\pi}{2} + b$ when $t = 0, \ x = 6$

$\therefore b = 2$ #

Question:

(a) $\int_0^1 \frac{4x}{2x+1}\,dx$

let $u = 2x+1$

$\therefore u-1 = 2x$

$1\,du = 2\,dx$

when $x=1$ $u=3$

$x=0$ $u=1$

① mark for correct manipulation.

$\therefore \int_0^1 \frac{2x}{2x+1} \cdot 2\,dx$

$= \int_1^3 \frac{u-1}{u} \cdot du$

① mark correct substitution of their variables

$= \int_1^3 \frac{u}{u} - \frac{1}{u} \cdot du$

$= \int_1^3 1 - \frac{1}{u}\,du$

$= \left. u - \ln u \right]_1^3$

① mark correct integration of their expression as long as the expression is not easier.

$= (3-\ln 3) - (1-\ln 1)$

$= 3 - \ln 3 - 1$

$= 2 - \ln 3.$

① mark correct answer only.

(b) $(3+11x)^{19} = (a+b)^n$

then $T_{k+1} = {}^nC_k\, a^{n-k} \cdot b^k$

(i) Show $\frac{T_{k+1}}{T_k} = \frac{11x(20-k)}{3k}$

Using the rule

$$\frac{T_{k+1}}{T_k} = \frac{n-k+1}{k} \cdot \frac{b}{a}$$

$$= \frac{19-k+1}{k} \cdot \frac{(11x)}{3}$$

① mark correct substitution into formula.

$$\therefore \frac{T_{k+1}}{T_k} = \frac{11x(20-k)}{3k}.$$

① mark correct manipulation

Question 7

(c) $(1+x)^{2n} + (1-x)^{2n} \qquad n > 0$

$$(1+x)^{2n} = (1)^{2n} + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \dots\dots x^{2n}.$$

$$(1-x)^{2n} = (1)^{2n} - {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \dots\dots (-x)^{2n}.$$

$$(1+x)^{2n} + (1-x)^{2n} = 2\times(1)^{2n} + 2\times {}^{2n}C_2 x^2 + \dots + 2\,{}^{2n}C_{2n}(x)^{2n}$$

$$= 2\left(1^{2n} + {}^{2n}C_2 x^2 + \dots {}^{2n}C_{2n}(x)^{2n}\right)$$

(1) mark correct answer in any form.

Evaluate

$$1 + {}^{20}C_2 + {}^{20}C_4 + \dots\dots {}^{20}C_{20}$$

$$= 1\left(1^{2n} + {}^{2n}C_2 x^2 + \dots {}^{2n}C_{2n} x^{2n}\right)$$

where $n = 10 \quad x = 1$ (1) mark correct substitution

$$= 1\left(1^{20} + {}^{20}C_2 1^2 + {}^{20}C_4 1^4 + \dots {}^{20}C_{20} 1^{20}\right)$$

$$= 1\left(1 + {}^{20}C_2 + {}^{20}C_4 + \dots {}^{20}C_{20}\right)$$

$$= \frac{(1+x)^{2n} + (1-x)^{2n}}{2} \quad \text{where } x = 1 \quad n = 10$$

$$= \frac{2^{20} + 0^{20}}{2} = 2^{19}$$ (1) mark correct answer Must show where the asnwer is derived from using (1)

$$= 524288$$

(i) without the rule: Using $T_{k+1} = {}^nC_k\, a^{n-k} \cdot b^k$

$$T_k = {}^nC_{k-1}\, a^{n-(k-1)} \cdot b^k$$

$$T_k = {}^nC_{k-1}\, a^{n-k+1} \cdot b^k.$$

$$T_{k+1} = {}^{19}C_k\, 3^{19-k} \cdot (11x)^k$$

$$T_k = {}^{19}C_{k-1} \cdot 3^{19-k+1} \cdot (11x)^{k-1}$$

$$= {}^{19}C_{k-1} \cdot 3^{20-k} \cdot (11x)^{k-1}$$ ① mark correct expression for T_k

$$\therefore \frac{T_{k+1}}{T_k} = \frac{{}^{19}C_k \cdot 3^{19-k} \cdot (11x)^k}{{}^{19}C_{k-1} \cdot 3^{20-k} \cdot (11x)^{k-1}}$$

$$= \frac{\frac{19!}{k!\,(19-k)!} \cdot 3^{19-k} \cdot (11x)^k}{\frac{19!}{(k-1)!\,(20-k)!} \cdot 3^{20-k} \cdot (11x)^{k-1}}$$

$$= \frac{(k-1)!\,(20-k)!\;11x}{k!\,(19-k)!\;3}$$

Working must be shown.

$$= \frac{(20-k) \cdot 11x}{k \cdot 3}$$ ① mark for correct

$$= \frac{11x(20-k)}{3k}$$

(ii) Greatest coefficient occurs when $\frac{T_{k+1}}{T_k} \geq 1$

$$\therefore \frac{11(20-k)}{3k} \geq 1$$ ① mark for their result from (i) without the x.

$$11(20-k) \geq 3k$$

$$220 - 11k \geq 3k$$

$$220 \geq 14k$$

$$14k \leq 220$$

$$k \leq 15\tfrac{5}{7}$$

$$\therefore k = 15$$

① mark correct term selected from their inequality for k.

$\therefore T_{k+1} > T_k$ when $k \leq 15\frac{5}{7}$

$\therefore T_{16} > T_{15}$

$\therefore T_{16}$ has the greatest coefficient $T_{16} = {}^{19}C_{15} \cdot 3^4 \cdot 11^{15}$

① mark correct answer.